NEO-SYLVESTER CONTRACTIONS AND THE SOLUTION OF SYSTEMS OF LINEAR EQUATIONS*

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1. *Introduction.* In 1851 Sylvester† stated, without proof, a theorem on contraction of determinants which had previously been stated by Hermite (1849)‡ for a special case. A proof of Sylvester's theorem was given by Studnička in 1879.§ A restatement of this theorem in a slightly different form permits significant applications to the determination of the ranks of matrices, the solution of systems of linear equations, and the calculation of partial and multiple coefficients of correlation.

2. *Restatement of Sylvester's Theorem.* The restatement of the theorem is as follows:

THEOREM 1. *In the nth order determinant,*

$$D \equiv |a_{ij}|, \qquad (i, j = 1, 2, \cdots, n),$$

let

$$M_{kl} \equiv \begin{vmatrix} a_{k_1l_1} & a_{k_1l_2} \cdots a_{k_1l_r} \\ a_{k_2l_1} & a_{k_2l_2} \cdots a_{k_2l_r} \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ a_{k_rl_1} & a_{k_rl_2} \cdots a_{k_rl_r} \end{vmatrix}$$

be a non-vanishing minor of order r, and associate with each element a_{ij} $(i \neq k, j \neq l)$ an $(r+1)$-rowed minor of D defined by the identity

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* Presented to the Society under another title, December 29, 1936.

† J. J. Sylvester, *On the relation between the minor determinants of linearly equivalent quadratic functions*, Philosophical Magazine, (4), vol. 1 (1851), pp. 295–305, 415.

‡ C. Hermite, *Sur une question relative à la théorie des nombres*, Journal de Mathématiques, vol. 14 (1849), pp. 21–30.

§ F. J. Studnička, *Ueber eine neue Determinantentransformation*, Sitzungsberichte Geschichte der Wissenschaften, vol. 9 (1879), pp. 487–494.