THE FOUR-VERTEX THEOREM FOR A CERTAIN TYPE OF SPACE CURVES.*

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It is the purpose of this note to establish for a certain type, A, of space curves the new form of the four-vertex theorem recently stated and proved for plane ovals.†

A space curve C shall be said to be of type A provided (a) it is a closed regular curve of class C'', (b) its curvature never vanishes, (c) the projection of its tangent indicatrix I on a plane π perpendicular to the line joining the origin O to the center of gravity G of I is an oval or, if G coincides with O, the projection of I on some plane, π, is an oval, and (d) this oval is traced just once when C is traced once.

By a vertex shall be meant a point, or an arc of constant curvature, for which the curvature has a relative extremum with respect to the neighboring arcs on either side. A vertex shall be said to be *primary* if the curvature at it has a maximum (minimum) which is greater (less) than the average curvature of the curve with respect to the arc. Otherwise, a vertex shall be termed *secondary*.‡

The theorem to be established may now be formulated as follows.

STATEMENT I. *On a curve of type A, whose curvature is not constant, there are at least four primary vertices. More precisely, the number of primary vertices, if finite, exceeds the number of secondary vertices by at least four, and is infinite if the number of secondary vertices is infinite.*

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† W. C. Graustein, *A new form of the four-vertex theorem*, Monatshefte für Mathematik und Physik, vol. 43 (1936), pp. 381–384; for a related theorem, see Hayashi, *Some general applications of Fourier series*, Rendiconti del Circolo Matematico di Palermo, vol. 50 (1926), p. 100. For other work on the four-vertex theorem in space see Süss, *Ein Vierscheitelsatz bei geschlossenen Raumkurven*, Tôhoku Mathematical Journal, vol. 29 (1928), pp. 359–362; Takasu, *Vierscheitelsatz für Raumkurven*, Tôhoku Mathematical Journal, vol. 39 (1934), pp. 292–298, and vol. 41 (1936), pp. 317, 318.

‡ Ibid., p. 381.