12. *Summary.* The results of §§2–7 can be summarized in the following theorem:

THEOREM. *The Kuratowski formula,*

$$\phi c \phi c \phi c \phi A = \phi c \phi A,$$

is satisfied for ϕ equal to any of the operators 1, c, d, e, i, j, f, k, and s.

Sections 10 and 11, together with this theorem, imply the following corollary:

COROLLARY. *The equation*

$$(\phi c)^{\alpha+2}\phi A = (\phi c)^{\alpha}\phi A$$

holds for every ordinal α equal to or greater than some finite or transfinite ordinal α_0, and for ϕ equal to any of the operators 1, c, d, e, i, h, j, b, f, k, and s.

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A CONDITION THAT A FIRST BOOLEAN FUNCTION VANISH WHENEVER A SECOND DOES NOT

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It is well known† that if two polynomials $f(x_1, \cdots, x_n)$ and $g(x_1, \cdots, x_n)$ in the field of complex numbers are such that f vanishes whenever g does not, then at least one of the two polynomials f and g is identically zero. The corresponding law, however, does not, in general, hold for Boolean functions, as may be seen by considering the two functions x and x' in a two-element Boolean algebra; the statement that either $x=0$ or else $x'=0$ in a two-element Boolean algebra is, indeed, the familiar law of excluded middle. It is the purpose of the present note to determine the conditions on the coefficients of two Boolean functions in order that the first vanish whenever the second does not.

The condition found involves *prime Boolean elements*, which are defined as follows:

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† See, for example, Bocher, *Introduction to Higher Algebra*, p. 8.