CYCLIC RELATIONS IN POINT SET THEORY*

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1. *Introduction.* The formula

$$(1) \qquad\qquad \phi c\phi c\phi c\phi A = \phi c\phi A,$$

where c denotes the operation of taking complements and ϕ is an arbitrary operator, is of considerable interest in the study of sets of points. Kuratowski† proved that the formula holds for a postulated closure or extension function. Zarycki‡ established the formula in case ϕA is the "interior" of A and Sanders§ established it for a general derived set operator satisfying the postulates:

I. $$d(A + B) = dA + dB,$$

II. $$d^2A \leqq dA.$$

In this paper we shall follow the established‖ practice of postulating a derived set operator d, subject to I and II, and, using a notation introduced by Chittenden, define certain terms as follows:

Identity: $1A = A$;

Complement: $cA = S - A$ (S denotes the entire space);

Extension:¶ $eA = A + dA$;

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* Presented to the Society, April 10, 1937.

† C. Kuratowski, *Sur l'opération Ā de l'analysis situs*, Fundamenta Mathematicae, vol. 3 (1922), pp. 182–199.

‡ M. Zarycki, *Notions fondamentales de l'analysis situs*, Fundamenta Mathematicae, vol. 9 (1927), pp. 3–15.

§ S. T. Sanders, Jr., *Derived sets and their complements*, this Bulletin, vol. 42 (1936), pp. 577–584.

‖ F. Riesz, *Stetigkeitsbegriff und abstrakte Mengenlehre*, Atti del 4 Congresso Internationale dei Matematici, Roma, 1910, vol. 2, p. 18; Chittenden, *On general topology and the relation of the properties of the class of all continuous functions to the properties of space*, Transactions of this Society, vol. 31 (1929), pp. 290–321.

¶ F. Hausdorff, *Mengenlehre*, pp. 109–129. Under the postulates given, the derived set corresponds to Hausdorff's set of β points, A_β. The extension corresponds to his α points, A_α. Similarly, hA corresponds to A_h, jA to A_j, bA to A_r (border is a translation of the German word "rand"), kA to A_k, and sA to A_s.