AN INDECOMPOSABLE LIMIT SUM

BY N. E. RUTT

It is the object of this paper to investigate a certain simple monotone sequence of continua. The theorem of the paper states conditions under which the limit sum of the sequence is indecomposable. The precise formulation and proof of the theorem will be undertaken after the following lemma is established.

LEMMA. *Let K be a plane bounded indecomposable continuum and L a plane bounded continuum such that $K\cdot L\neq 0$, and that $c(L)$* includes a particular component λ containing the component δ of $c(L+K)$ with the following properties*:

(a) *the set L contains two distinct points, a and c, connected through δ by the arc B which divides δ into δ_i and δ_e, and λ into λ_i and λ_e*;

(b) *both λ_i and λ_e contain points of K.*

Then each component of $c(K+L)$ has as its boundary a proper subset of $K+L$.

The assumption that $c(K+L)$ has a component γ with boundary Γ such that $\Gamma\supset(K+L)$ will be shown contradictory. Let the boundaries of δ_i, δ_e, λ_i, λ_e be respectively Δ_i, Δ_e, Λ_i, and Λ_e. Suppose that δ is unbounded and also δ_e and λ_e, so that δ_i and λ_i will necessarily be bounded. Evidently $\lambda_i\supset\delta_i$ and $\lambda_e\supset\delta_e$. Consider first the case in which L is irreducible between a and c.

Both Λ_i and Λ_e contain L. For $\Lambda_i\subset L+B$ and $\Lambda_e\subset L+B$; so, since B is an arc with $L\cdot(B)$† $=0$, $\Lambda_i\cdot L$ and $\Lambda_e\cdot L$ are continua containing $a+c$. If either of these is not identical with L, then L is reducible between a and c. The domains δ and γ are, moreover, identical, for both λ_i and λ_e contain points of K, therefore points of Γ, and therefore points of γ. There is thus an arc X in γ such that $X\cdot\lambda_i\neq 0$ and $X\cdot\lambda_e\neq 0$, and since $X\cdot L=0$, then $(B)\cdot X\neq 0$. This implies $X\cdot\delta\neq 0$, accordingly $\gamma\cdot\delta\neq 0$; and as both γ and δ are components of $c(K+L)$, then $\gamma=\delta$, and $\lambda+L\supset\Gamma\supset K$.

Let K_i be the sum of $K\cdot\lambda_i$ and of all the components of $L\cdot K$

* If X is a point set then $c(X)$ is the complement of X.

† If X is an arc then (X) is X with ends omitted.