CONCERNING NORMAL AND COMPLETELY NORMAL SPACES*

BY F. B. JONES

Urysohn has shown that any completely separable, normal topological space is metric. It is the principal object of this paper to establish a similar result for certain separable spaces.

THEOREM 1. *Every subset of power c*† *of a separable normal*‡ *Fréchet space-L (or -H) has a limit point.*

PROOF. Suppose, on the contrary, that S is a separable, normal Fréchet space-L (or -H) which contains a point set M of power c having no limit point. Let Z denote a countable subset of S such that every point of S either belongs to Z or is a limit point of Z. Since S is normal, there exists for each proper subset J of M a domain D_J which contains J but which neither contains a point of $M-J$ nor has a limit point in $M-J$. If J and K are two different proper subsets of M, then $Z \cdot D_J$ and $Z \cdot D_K$ are different subsets of Z. Hence, there are at least as many subsets of Z as there are proper subsets of M. However, since M is of power c and Z is only countable, there are *more* than c proper subsets of M but at most c subsets of Z. This is a contradiction.

The above argument with slight changes establishes the following three theorems.

THEOREM 2. *Every subset of power c of a separable, completely normal*§ *Fréchet space-L (or -H) contains a limit point of itself.*

THEOREM 3. *If* $2^{\aleph_1} > 2^{\aleph_0}$, *every uncountable subset of a separable normal Fréchet space-L (or -H) has a limit point.*||

* Presented to the Society, October 28, 1933.

† The number c is the power of the continuum.

‡ A space is said to be *normal* provided that, if P and Q are two mutually exclusive closed sets, there exist two mutually exclusive domains containing P and Q respectively.

§ A space is said to be *completely normal* provided that, if P and Q are two mutually separate point sets, there exist two mutually exclusive domains containing P and Q respectively.

|| The numbers $\aleph_0$ and $\aleph_1$ are the first and second transfinite cardinals respectively. That $2^{\aleph_1} > 2^{\aleph_0}$ is an immediate consequence of a well known theorem if the *hypothesis of the continuum* holds true, that is, if $\aleph_1 = c$.