p-ALGEBRAS OF EXPONENT p*

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A. A. Albert and O. Teichmüller have recently investigated the structure of p-algebras, that is, normal simple algebras of degree p^e and characteristic p.‡ In particular they showed that a necessary and sufficient condition that such an algebra have exponent p is that it be similar to an algebra A having a maximal subfield $C=F(c_1, c_2, \cdots, c_m)$, where $c_i^p=\gamma_i \epsilon F$, the underlying field. The latter algebra is cyclic. It is the purpose of this note to apply some results of my paper *Abstract derivation and Lie algebras*§ to obtain a new generation of A. For $m=1$ this generation is more symmetric than the cyclic generation. We obtain a condition that A be a matrix algebra in terms of the new generation, and when $m=1$ we have as a consequence a reciprocity law for fields of characteristic p.

Let A be a normal simple algebra of degree p^m (order p^{2m}) over a field F of characteristic p and suppose A contains the maximal subfield $C=F(c_1, c_2, \cdots, c_m)$, $c_i^p=\gamma_i \epsilon F$. Let D be an arbitrary derivation of C over F, that is, a mapping $x \rightarrow xD$ of C into itself such that

$$(x+y)D = xD + yD, \qquad (x\alpha)D = (xD)\alpha,$$
$$(xy)D = (xD)y + x(yD), \qquad \alpha \epsilon F.$$

It is known that D may be chosen so that the only elements z such that $zD=0$ are those of F,‖ and for a D of this type I have shown that

$$(1) \qquad x(D^{p^m} + D^{p^{m-1}}\tau_1 + \cdots + D\tau_m) = 0, \qquad \tau_i \epsilon F,$$

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‡ A. A. Albert, *On normal division algebras of degree p^e over F of characteristic p*, Transactions of this Society, vol. 39 (1936), pp. 183–188, and *Simple algebras of degree p^e over a centrum of characteristic p*, Transactions of this Society, vol. 40 (1936), pp. 112–126. O. Teichmüller, *p Algebren*, Deutsche Mathematik, vol. 1 (1936), pp. 362–388.

§ Transactions of this Society, vol. 42 (1937), pp. 206–224, referred to as J.

‖ R. Baer, *Algebraische Theorie der differentierbaren Funktionenkörper*. I, Sitzungsberichte Heidelberger Akademie, 1927, pp. 15–32.