BOUNDARY VALUE PROBLEMS OF THE CALCULUS OF VARIATIONS†

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1. *Introduction.* The term boundary value problem is applied to the question of determining whether a given system of differential equations (in general, a given system of functional equations) has one or more solutions satisfying certain prescribed end or boundary conditions; and, if so, the determination of the character of these solutions and how their character changes when the differential equations or boundary conditions change. This address is restricted to the discussion of a class of linear boundary problems which are intimately associated with the calculus of variations. These boundary value problems have been used in establishing sufficient conditions, especially for the more complicated problems of the calculus of variations. On the other hand, the principles and theorems of the calculus of variations have been of extreme significance in the advancement of the theory of such boundary value problems. In fact, the calculus of variations has served to unify a certain class of boundary problems much larger than that seemingly represented by the problem that we shall first formulate. In view of the rather extensive interest and study of these problems within recent years, it seems proper at this time to discuss the present status of such boundary problems, to compare the various methods that have been used in their treatment, and to indicate various questions concerning them that are as yet unsolved.

Historically, the study of boundary problems associated with a second order linear differential equation dates from the time of Euler and D'Alembert. The first somewhat general theory of such problems, however, is that given by Sturm [1]‡ in his fundamental memoir of 1836. One of the most important questions for the more complicated boundary problems that we shall here consider is that of generalizing the Sturm oscillation and

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‡ Numerals in square brackets refer to the bibliography at the end of the paper.