Then

$$\frac{F_{n,m}(x)}{[\theta_k(x)]^n}$$

$$= \frac{(x - a_1) \cdots (x - a_{m-1})(x - a_{m+1}) \cdots (x - a_\nu) G_{n,m}(x)}{(a_m - a_1) \cdots (a_m - a_{m-1})(a_m - a_{m+1}) \cdots (a_m - a_\nu) G_{n,m}(a_m)}.$$

From this relation it is easily seen that all the conditions of Corollary 2 are satisfied, and our statement follows.

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DUALISM IN ABELIAN GROUPS*

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It has been proved† that a finite Abelian group contains as many subgroups of a given order n as it contains factor groups of order n ($=$subgroups of index n) and E. Steinitz‡ knew that a finite Abelian group contains as many subgroups of a given structure $\mathbf{n}$ as it contains factor groups of structure $\mathbf{n}$. It is the aim of this note to prove that such a *dualism* exists in the Abelian group G if, and only if, G is a group without elements of infinite order whose primary components are finite. This is remarkable as an exception to the rule that every proposition which is satisfied in finite Abelian groups holds also true in every primary Abelian group such that the orders of its elements are bounded.

Let G and G' be two (additively written Abelian) groups. Then the function $\mathbf{d}$ is a dualism of G upon G' if it has the following properties:

(1) $\mathbf{d}$ is defined for every subgroup S of G and $S\mathbf{d}$ is a uniquely determined subgroup of G';

(2) to every subgroup S' of G' there exists a subgroup S of G such that $S\mathbf{d} = S'$;

(3) $S \leqq T$ ($\leqq G$) if, and only if, $T\mathbf{d} \leqq S\mathbf{d}$ ($\leqq G'$);

* Presented to the Society, October 31, 1936.

† Garrett Birkhoff, *Subgroups of Abelian groups*, Proceedings of the London Mathematical Society, (2), vol. 38 (1934), pp. 385–401.

‡ E. Steinitz, Jahresberichte der Deutschen Mathematiker-Vereinigung, vol. 9 (1901), pp. 80–85.