A CONDITION IN INVARIANT FORM FOR A NET WITHOUT DETOURS*

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Suppose that on a surface whose linear element is given by

$$ds^2 = Edu^2 + 2Fdudv + Gdv^2, \qquad D^2 = EG - F^2 \neq 0,$$

we have a system of curves, one and only one curve of each family passing through each point of that region of the surface under consideration, and distances along these curves being measured positively in a direction preassigned for each family. If this system of curves possesses the property that the distance between any two points of the region, measured along curves of the system according to the preassigned laws of orientation, is independent of the path chosen, then the system is called a *net without detours* (Kurvennetz ohne Umwege).

A net without detours being given, the distance along curves of the net from a fixed point O to a variable point $P:(u, v)$ must be a function of position only. If the function $f(u, v)$ is given, and we wish to find the net without detours corresponding to it, we must have for the curves of the net $df^2 = ds^2$, or,

$$(f_u du + f_v dv)^2 = Edu^2 + 2Fdudv + Gdv^2,$$

or,

$$(E - f_u^2)du^2 + 2(F - f_u f_v)dudv + (G - f_v^2)dv^2 = 0. \tag{1}$$

This differential equation defines the net without detours corresponding to $f(u, v)$.

If a family of curves $\phi(u, v) = \text{const.}$ is to be one family of a net without detours for a given distance function $f(u, v)$, then $(\phi_u du + \phi_v dv)$ must be a factor of the left side of (1), so that their Sylvester eliminant must vanish:

$$\begin{vmatrix} E - f_u^2 & 2(F - f_u f_v) & G - f_v^2 \\ \phi_u & \phi_v & 0 \\ 0 & \phi_u & \phi_v \end{vmatrix} = 0,$$

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