PRODUCTS OF NÖRLUND TRANSFORMATIONS*

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1. *Introduction.* For certain purposes the *symmetric* product of two Nörlund transformations, to be presently defined, has been found to be more convenient than the ordinary product. The aim of this paper is to compare the fields of convergence of these two products. Let M be the ordinary arithmetic mean and P any Nörlund transformation. The principal results obtained in this paper are *first*, that the field of convergence of the ordinary product includes that of the symmetric product; *secondly*, that the converse is in general not true; and *thirdly*, that a necessary and sufficient condition for the equivalence of these products can be stated.

2. *Permutability with M.* A Nörlund transformation [1]† is a special case of that corresponding to a triangular matrix

$$(1) \qquad a_{nk} = \frac{p_{n-k}}{P_n}, \qquad (n \geqq k \geqq 0;\ P_n = p_0 + p_1 + \cdots + p_n),$$

where p_n is a sequence of positive numbers. If

$$(2) \qquad \frac{p_n}{P_n} \to 0,$$

then the matrix a_{nk} is regular. We may assume without loss of generality that $p_0 = 1$. For if $p_0 = 0$, we may take

$$y_{n+1} = \frac{p_0 x_{n+1} + p_1 x_n + \cdots + p_{n+1} x_0}{p_0 + p_1 + \cdots + p_{n+1}} = \frac{p_1 x_n + \cdots + p_{n+1} x_0}{p_1 + p_2 + \cdots + p_{n+1}},$$

which is the result of applying the transformation corresponding to the sequence p_{n+1} to the sequence x_n. If now $p_0 \neq 0$, we may obviously choose $p_0 = 1$.

It has been shown [1], [3], that all Nörlund transformations are consistent. M. Riesz [2] has given necessary and sufficient

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† Such bold-faced numerals refer to the bibliography at the end of this paper.