CURVES BELONGING TO PENCILS OF LINEAR LINE COMPLEXES IN S_4

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1. *Introduction.* It has been demonstrated in at least two ways* that every curve in S_3, whose tangents belong to a non-special linear line complex can be mapped into a curve in S_3 all of whose tangents meet a fixed conic. In this paper, similar theorems are obtained for curves in S_4 whose tangents belong to (1) a single linear complex, (2) a pencil of linear complexes.

In what follows we shall use the symbol Γ to represent a non-special complex, that is, a complex which does not consist of the totality of lines which meet a plane. We shall use the symbol Π to represent a pencil of complexes which does not contain any special complexes. The customary symbol V_m^r will be used to represent a variety of order r and of dimension m.

2. *Hyperpencil of Lines.* We note first that no curve lying in S_4 but in no linear subspace of S_4 can belong to a special complex. For all the tangents of such a curve would have to meet the singular plane of the complex, which would require the osculating S_3's of the curve to contain the plane. This is impossible unless the curve lies entirely in an S_3 containing the singular plane. We are thus concerned with non-special complexes in (1) and with pencils which contain no special complexes in (2).

Through an arbitrary point of S_4 pass ∞^2 lines belonging to a non-special complex Γ. These lines lie in an S_3, the polar S_3 of the point as to Γ, and form what we shall call a hyperpencil of lines. For every complex Γ, there is a unique point with the property that every line which passes through that point belongs to Γ. We shall call this point the vertex of Γ. Of the five types of pencils of complexes in S_4 all but one contain special complexes. The one admissible type, Π, consists of ∞^1 complexes whose vertices lie on a non-composite conic, K. Through an ar-

* V. Snyder, *Twisted curves whose tangents belong to a linear complex*, American Journal of Mathematics, vol. 29 (1907), pp. 279–288.

C. R. Wylie, Jr., *Space curves belonging to a non-special linear line complex*, American Journal of Mathematics, vol. 57 (1935), pp. 937–942.