As examples we mention $y^3-x^4=0$, $y^3-x^5=0$, and $[y-1+(1-x^2)^{1/2}]^2-x^5=0$ with the determination of $(1-x^2)^{1/2}$ which equals 1 when $x=0$. These give respectively, at the origin, a minimum, a point of inflection, and a cusp with both branches concave upward. In none of the three cases is y analytic in x at the origin. An example where the locus is a single point is given by $y+ix=0$.

In the case of a reducible function $f(x, y)$, the real locus $f(x, y)=0$ neighboring (x_0, y_0) consists of a finite number of configurations of the kind described in the theorem, no two of which have any point except (x_0, y_0) in common. This is easily proved by use of theorems on resultants and on divisibility of one function by another. Of course two irreducible factors may have exactly the same locus.

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A PARTIAL DIFFERENTIAL EQUATION CONNECTED WITH THE FUNCTIONS OF THE PARABOLIC CYLINDER*

BY HARRY BATEMAN

The partial differential equation

$$\sum_{s=1}^{p}\left(\frac{\partial^2 V}{\partial x_s^2} - x_s\frac{\partial V}{\partial x_s}\right) + \nu V = 0, \tag{1}$$

which was considered by Mehler† in 1866, is a slight modification of an equation which occurs in wave-mechanics in the theory of the rotator in a plane and in space.‡ The case in which ν is a positive integer is then of chief physical interest and Mehler's simple solution

$$V = \prod_{s=1}^{p} H_{m_s}(x_s), \qquad \sum_{s=1}^{p} m_s = \nu, \tag{2}$$

acquires a physical significance. The function $H_m(x)$ is the polynomial of Laplace and Hermite defined by the equation

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† F. G. Mehler, Journal für Mathematik, vol. 66 (1866), p. 161.

‡ A. Sommerfeld, *Atombau und Spektrallinien, wellenmechanischer Ergänzungsband*, 1929, p. 23.