ON CONTINUED FRACTIONS OF THE FORM

$1 + \overset{\infty}{\underset{1}{K}} (b_\nu z/1)$

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1. *Introduction.* The principal object of this paper is to determine the region of convergence of the infinite continued fraction

$$(1)\qquad 1 + \overset{\infty}{\underset{1}{K}} (b_\nu z/1) = 1 + \frac{b_1 z}{1} + \frac{b_2 z}{1} + \cdots, \qquad (b_n \neq 0),$$

when $b_1, b_2, b_3, \cdots$ are real or complex numbers such that for some $k \geqq 1$

$$(2)\qquad \lim_{n=\infty} b_{nk+m} = \sigma_m, \qquad (m = 1, 2, 3, \cdots, k).$$

The results may be stated in terms of the numerators and denominators $u_{n,\lambda}$, $v_{n,\lambda}$ of the nth convergent of the continued fraction $1+K_{\nu=1}^{\infty}(\sigma_{\nu+\lambda}z/1)$, $(\sigma_{nk+m}=\sigma_m)$, as follows.

THEOREM 1. *Let*† *us write* $G_k = v_{k-1,1}v_{k-1,2}\cdots v_{k-1,k}$ *and* $H_k = v_k + u_{k-1} - v_{k-1}$; *and let us set*

$$Z(z) = -(-z)^k \sigma_1\sigma_2\cdots\sigma_k/H_k^2 .$$

Let R be an arbitrary bounded closed and connected region of the z plane containing the origin on the interior and which contains (within or upon the boundary) none of the zeros of the polynomials G_k, H_k, nor points z such that $Z(z)$ is a real number $\leqq -1/4$. Then (1) converges over R except at certain isolated points $p_1, p_2, \cdots, p_\mu$, and uniformly over the region obtained from R by removing the interiors of small circles with centers $p_1, p_2, \cdots, p_\mu$. The limit is a non-rational function of z analytic over R except at $p_1, p_2, \cdots, p_\mu$, which are poles.

The function $Z(z)$ determines a transformation of the z plane into the Z plane and $Z = Z(z)$. Except in the case $\sigma_1\sigma_2\cdots\sigma_k = 0$, the set of points in the z plane such that Z is real and

† We write $u_{n,0} = u_n$, and $v_{n,0} = v_n$.