

no point of B . Let $f(z)$ be analytic and bounded in G . A necessary and sufficient condition for the existence of polynomials $\{p_n(z)\}$ which converge to $f(z)$ in G so that (1) holds is that there exist a function analytic and bounded in Γ and equal to $f(z)$ in G .

The proof of this theorem is much the same as for Theorem A taken together with the remark of §5 in the earlier paper and is therefore omitted.

The conclusion of Theorem D simply means of course that $f(z)$ shall be analytically extensible throughout Γ .

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A GENERALIZED PARSEVAL'S RELATION

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A function $\phi(x)$ which is non-negative, convex, and satisfies the conditions $\phi(0) = 0$ and $(\phi(x)/x) \rightarrow \infty$ as $x \rightarrow \infty$ will be called a Young's function. Given such a function $\phi(x)$, a second function, $\psi(x)$, with the same properties can be found such that Young's inequality, $ab \leq \phi(a) + \psi(b)$, holds for every $a, b \geq 0$. The functions $\phi(x)$ and $\psi(x)$ are then said to be complementary in the sense of Young.†

If $x(t)$ is such that $\int_a^b \phi(|x|) dt$ exists, $x(t)$ is said to belong to the space $L_\phi(a, b)$. This space is not necessarily linear.‡ For this reason we denote by $L_\phi^*(a, b)$ the class of all functions $x(t)$, $a \leq t \leq b$, such that the product $x(t)y(t)$ is integrable for every $y(t) \in L_\psi(a, b)$. If we put

$$\|x\|_\phi = \sup_y \left| \int_a^b x(t)y(t) dt \right|$$

for all $y(t)$ with

$$\rho_y \equiv \int_a^b \psi(|y|) dt \leq 1,$$

then L_ϕ^* is a linear metric, and complete space.§ A function

† W. H. Young, Proceedings Royal Society, (A), vol. 87 (1912), pp. 225-229.

‡ W. Orlicz, Über eine gewisse Klasse von Räumen vom Typus B, Bulletin, Académie Polonaise, (A), Cracovie (1932).

§ A. Zygmund, Trigonometrical Series, 1935, pp. 95-100.