THE CONSTANTS IN WARING'S PROBLEM FOR ODD POWERS

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1. *Introduction.* In a previous paper* the writer determined the constants in the Hardy-Littlewood analysis of Waring's problem. It was then possible to obtain new universal theorems. The proof depended on the following results.

THEOREM A. (Theorem 46, page 429, and (C), page 442 in paper T.) *Let $k \geqq 6$ be an integer; s an integer; $a = 1/k$;*

$$d = [\log (k - 1)/\log 2]; \quad D = (d + 2)(k - 1) - 2^{d+1} + 0.1;$$

$$s_2 = 4 + \zeta_k = 4 + \left[\frac{(k - 2) \log 2 - \log k + \log (k - 2)}{\log k - \log (k - 1)}\right];$$

$$\eta = \frac{D(s - 2) + k2^{k-2}(1 + (1 - a)^{s_2 - 2})}{(s - 2) - (k - 2)2^{k-2} - k}.$$

Then every integer $n > C$, where $\log_e C = 20k^3 2^\eta$, is a sum of $s + s_2$ kth powers.

The proof of universal theorems was greatly simplified by Dickson's new method. Using Theorem A and very short tables he proved that† *every* integer is a sum of 259 seventh powers, 575 eighth powers, 981 ninth powers, 10711 twelfth powers, $\cdots$. All these results are considerably better than those obtained by algebraic methods.

In another paper‡ the writer obtained new asymptotic results for odd powers. It is the purpose of the present note to point out that the constants of this paper may also be evaluated by the methods of paper T. This leads to still better universal

* Transactions of this Society, vol. 36 (1934), pp. 395–444. This will be referred to as paper T.

† This Bulletin, vol. 39 (1933), Theorems 15–18, pp. 713–714.

‡ Proceedings of the London Mathematical Society, (2), vol. 37 (1934), pp. 257–291. This will be referred to as paper P.