A NEW UNIVERSAL WARING THEOREM FOR EIGHTH POWERS

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1. *Introduction.* Hardy and Littlewood* in their proof of Waring's theorem obtained a constant $C=C(s, k)$ beyond which every number is a sum of s integral kth powers $\geqq 0$. Recently Dickson perfected an algebraic method by which he was able to show that every positive integer $\leqq C$ is a sum of s integral kth powers $\geqq 0$. Thus we are now able to obtain universal Waring theorems for relatively small values of s.

We shall consider in this paper the problem of meeting the Hardy and Littlewood constant by Dickson's method and establishing a new universal Waring theorem for eighth powers. The earlier result for eighth powers was 575, obtained by Dickson.†

2. *Proof of the Principal Theorem.* We write

$$(1) \qquad a = 2^8, \qquad b = 3^8, \qquad c = 4^8, \qquad d = 5^8, \qquad e = 6^8.$$

The right side of

$$m = n + Aa + Bb + \cdots + Qq, \qquad (n, A, B, \cdots, Q \text{ integral}),$$

is a *resolution* of m of *weight* $w(m)=n+A+B+\cdots+Q$. When $n, A, B, \cdots, Q\geqq 0$ the resolution is a *decomposition*.

By division we obtain

$$(2)\ b = 161 + 25a,\ c = -74 + 10b,\ d = 56 + 15a + 9b + 5c,$$

$$(3) \qquad e = 21 + 22a + 7b + c + 4d.$$

Consider an integer M, such that $2d+e\leqq M\leqq 3d+e$. We can express the integer $P=M-2d-e$ uniquely in the form $R+N$, where

$$(4) \qquad 0 \leqq R < a = 256, \qquad N = Aa + Bb + Cc,$$

$$(5)\ C = [P/c],‡\ B = [(P - Cc)/b],\ A = [(P - Bb - Cc)/a].$$

* A simplified proof can be found in Landau, *Vorlesungen über Zahlentheorie*, vol. 1, 1927, pp. 235–360.

† This Bulletin, vol. 39 (1933), p. 713.

‡ $[x]$ denotes the largest integer $\leqq x$.