ON A THEOREM OF PLESSNER

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Plessner‡ has shown that if $f(x) \subset L_2$ on $(-\pi, \pi)$ and

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx),$$

then

$$\sum_{n=2}^{\infty} (a_n \cos nx + b_n \sin nx)(\log n)^{-1/2} \tag{1}$$

converges almost everywhere on $(-\pi, \pi)$. We designate the set where (1) converges by $E(Pl, f)$. This set is then known to be of measure 2π. The sets $E(F, f)$, consisting of the points where

$$\phi(t) = f(x + t) + f(x - t) - 2f(x) \to 0, \text{ as } t \to 0,$$

and $E(L, f)$, consisting of the points where

$$\Phi(t) = \int_0^t | \phi(\tau) | \, d\tau = o(t), \text{ as } t \to 0,$$

are of much importance in the theory of Fourier series. The set $E(L, f)$ is known to be of measure 2π for all integrable functions. It is obvious that

$$E(F, t) \subset E(L, f).$$

We propose in this note to investigate the inclusion relationships between these sets and $E(Pl, f)$. We shall prove

$$E(F, f) \not\subset E(Pl, f), \tag{2}$$

and

$$E(Pl, f) \not\subset E(L, f). \tag{3}$$

We first consider (2). Plessner§ showed that, if (1) converges,

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‡ A. Plessner, Journal für Mathematik, vol. 155 (1926), pp. 15–25.

§ Loc. cit., p. 22.