TOPICS IN THE FUNCTIONAL CALCULUS†

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PART I. THE THEORY OF FUNCTIONALS

In this lecture it is proposed to outline an abstract theory of functionals, with a development paralleling that of the theory of functions of real variables, and including also a chapter on analytic functionals. In Part II, some applications of the general theory to various sorts of equations are indicated.

From the abstract point of view, the functional calculus is a form of general analysis, and as such it was effectually initiated by Fréchet's thesis in 1906. Since then a large number of researches have been concerned with the topological properties of abstract sets, and with the properties of continuous or semicontinuous functionals.

The postulational basis for an abstract topological theory may take various forms. A general basis consists of a general, that is, unrestricted class $\mathfrak{X}$ of elements x, and an unrestricted function K on $\mathfrak{S}$ to $\mathfrak{S}$, where $\mathfrak{S}$ is the class of all subsets E of $\mathfrak{X}$. The function K is then a set-valued function of sets. The system $(\mathfrak{X}, K)$ constitutes a topological space. It has been shown by Chittenden [7, pp. 294–298]‡ that a related set-function H may always be defined such that the space $(\mathfrak{X}, H)$ has the three properties:

I. $H(D+E)=H(D)+H(E)$.

II. For every set E, $H(E)$ contains $H(H(E))$.

III. If E is finite, $H(E)$ is null.

Such a space $(\mathfrak{X}, H)$ is called an *accessible space* by Fréchet.

If the points of $H(E)$ are called the *points of accumulation* of the set E, then closed sets may be defined as usual. A point of a set E is interior to E in case it is not a point of accumulation of the complement of E. Open sets are those consisting only of interior points. The neighborhoods of a point x may be defined as those sets having x as an interior point. A set E is called *com-*

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† An address delivered by invitation of the program committee at the Chicago meeting of this Society, April 19, 1935.

‡ References in brackets are to the bibliography at the end of the paper.