$$[|a_{p_i}| - \alpha^{q_i}] - \left[\frac{k^{p_i}}{1-k} + |\epsilon_{p_i}||a_{p_i}| + \epsilon'_{p_i}\frac{\beta^{p_i+1}}{1-\beta}\right]$$

$$= |a_{p_i}|(1 - |\epsilon_{p_i}|) - \alpha^{q_i} - \frac{k^{p_i}}{1-k} - \epsilon'_{p_i}\frac{\beta^{p_i+1}}{1-\beta}$$

$$> \beta^{p_i}(1 - |\epsilon_{p_i}|) - \alpha^{q_i} - \frac{k^{p_i}}{1-k} - \epsilon'_{p_i}\frac{\beta^{p_i+1}}{1-\beta}$$

$$= \beta^{p_i}\left(1 - |\epsilon_{p_i}| - \frac{\alpha^{q_i}}{\beta^{p_i}} - \frac{k^{p_i}}{\beta^{p_i}(1-k)} - \epsilon'_{p_i}\frac{\beta}{1-\beta}\right).$$

Now for $i$ sufficiently large all the terms within the last parentheses except the first are as small as we please. Hence for sufficiently large $i$ the difference in question is positive. From this contradiction the theorem follows.

In conclusion, we may note as a simple corollary of the above theorem that if $\overline{\lim}_{n\to\infty}|a_n|^{1/n}=1$, then $\lim_{n\to\infty}|a_{n+1}/a_n|=1$ if and only if there exists a sequence of *real* numbers $\lambda_n$ such that $\lim_{n\to\infty}\lambda_n=1$ and $\overline{\lim}_{n\to\infty}||a_{n+1}|-\lambda_n|a_n||^{1/n}<1$.

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# ON THE COEFFICIENTS OF A TYPICALLY-REAL FUNCTION*

BY M. S. ROBERTSON†

1. *Introduction.* It is well known‡ that if

$$f(z) = \sum_{n=0}^{\infty} a_n z^n \tag{1}$$

is regular for $|z| \leqq 1$, and if $E$ is defined by the formula

$$E \equiv \underset{|z_1|=|z_2|=1}{\text{maximum}} |\mathcal{R}f(z_1) - \mathcal{R}f(z_2)|, \tag{2}$$

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‡ See E. Landau, Archiv der Mathematik und Physik, (3), vol. 11 (1906), pp. 31–36.