THE BLOCH CONSTANT $\mathfrak{A}$ FOR A SCHLICHT FUNCTION*

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The following theorem† is due to Bloch.

There is an absolute positive constant P with the following property. Let $f(x)$ be regular for $|x|<1, f'(0)=1$. Then $y=f(x)$ maps the circle $|x|<1$ on a region (in a Riemann surface over the y plane) containing a circle of radius P in a single sheet.

(Without the condition $f'(0)=1$ there is a circle of radius $P|f'(0)|$.)

Landau‡ defines three absolute constants connected with this theorem. $\mathfrak{B}$ is the upper bound of the P which satisfy the theorem as stated. $\mathfrak{L}$ is the upper bound of the P if we require only that there be a circle of radius P in the y plane each point of which is covered by some sheet of the map. $\mathfrak{A}$ is the corresponding bound if $f(x)$ is given as schlicht (that is, $f(x_1)\neq f(x_2)$ for $x_1\neq x_2$, so that the map has only one sheet).

We have clearly $\mathfrak{B}\leqq\mathfrak{L}\leqq\mathfrak{A}$. The chief object of the paper by Landau is to give as close lower and upper bounds as possible for these three constants. He proves $0.39<\mathfrak{B}<0.56$, $0.43<\mathfrak{L}<0.56$, and $\mathfrak{A}>0.56$ (so that $\mathfrak{L}<\mathfrak{A}$). However, as an upper bound for $\mathfrak{A}$ he obtains no new result, but mentions a result of Szegö $\mathfrak{A}\leqq\pi/4$, as the best result which he knows. This follows from consideration of the function

$$y=\frac{1}{2}\log\frac{1+x}{1-x}=x+\cdots,$$

which maps the circle $|x|<1$ on the strip $|\mathfrak{J}y|<\pi/4$.

A better bound than $\pi/4$ can be obtained by mapping $|x|<1$ on as simple a region as a circle slit along a radius from the center to the circumference (Theorem 1). Still better bounds may be

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† For a proof and further references, see a paper by Landau, *Über die Blochsche Konstante und zwei verwandte Weltkonstanten*, Mathematische Zeitschrift, vol. 30 (1929), pp. 608–634.

‡ Loc. cit.