SPACE INVOLUTORIAL TRANSFORMATIONS OF THE GEISER AND BERTINI TYPES*

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1. *Introduction.* One form of generalization of a plane involution is a space involutorial transformation in which each plane of a pencil is invariant and in each such plane there is a plane involution of the same type. Particular examples of this for Geiser and Bertini involutions have been given by Carroll,† Snyder and Lehr,‡ and Sharpe and Dye.§ I shall discuss a more general form for space involutorial transformations arising from these plane involutions by means of a mapping on a cubic surface.|| The Bertini transformation obtained has the signature

$$I_{120n+51}: l^{120n+34+6t} + (O, \bar{O})^{120n+40} + C^{6}_{12n+6};$$

the Geiser transformation has the signature

$$I_{24n+19}: l^{24n+11+3t} + O^{24n+14} + C^{3}_{12n+6}.$$

2. *The Geiser Transformation.* In an involutorial space transformation of the Geiser type, let $x_4 = \lambda x_3$ be the equation of the invariant pencil of planes, and let I_G be the Geiser involution in the plane $x_4 = \lambda x_3$. Choose one of the fundamental points of I_G as $O \equiv (1, \lambda, 0, 0)$ on the line $l \equiv x_3 = x_4 = 0$, and map the I_G on a cubic surface F_3 by means of the bilinear $T_{3\text{-}3}$ defined by the matrices

$$\|(a_{i1}y_i) \quad (a_{i2}y_i) \quad (a_{i3}y_i) \quad (a_{i4}y_i)\|, \tag{1}$$

$$\|(a_{1i}x_i) \quad (a_{2i}x_i) \quad (a_{3i}x_i) \quad (a_{4i}x_i)\|, \tag{2}$$

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† E. T. Carroll, American Journal of Mathematics, vol. 54 (1931), pp. 707–717, and vol. 56 (1934), pp. 96–108.

‡ V. Snyder and M. Lehr, American Journal of Mathematics, vol. 53 (1931), pp. 186–195.

§ F. R. Sharpe and L. A. Dye, Transactions of this Society, vol. 36 (1934), pp. 292–305.

|| For a discussion of the mapping of a Geiser or a Bertini plane involution on a cubic surface, see H. F. Baker, *Principles of Geometry*, vol. 6, pp. 122–130.