ON THE LIMIT OF A SEQUENCE OF POINT SETS

BY HSÜ PAO-LU

A variable point P_n is said to approach the point P as its limit if to an arbitrary positive ϵ there corresponds an m such that

$$\overline{P_nP} < \epsilon, \qquad (n > m).$$

In other words, P is to have the property that every neighborhood of it contains almost all* the points P_n.

In attempting to generalize this definition to a sequence of point sets $M_1, M_2, \cdots$, one is naturally led to begin with a definition of the neighborhood of a set and then write down (Definition A_0) the last sentence of the last paragraph, replacing the letter P by M.

DEFINITION. *By the ϵ-neighborhood of a set M is meant the set of all points which have a distance $<\epsilon$ from some point of M. We shall denote it by $(\epsilon)_M$.*

DEFINITION A_0. *A point set M is called a limit of the sequence of sets $M_1, M_2, \cdots$, if every neighborhood of it contains almost all the sets M_i as partial sets.*

But the above definition is far from being useful, because the limit would then not be unique. In the first place, if the set M is a limit in the sense of Definition A_0, and if M has a cluster point C, then the set $M-C$ has also the property of being a limit of the sequence. Secondly every set containing M as a partial set is a fortiori a limit.

The first difficulty is overcome by requiring M to be closed, and the second difficulty is met by adding still another condition (γ):

DEFINITION A. *A set M is said to be the limit of the sequence of sets $M_1, M_2, \cdots$, if it has the following properties:*

(α) *M is closed.*

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* Thereby is meant that at most a finite number of the points P_i can lie outside the neighborhood.