

erally separable domains of diameter $1/n$ or less. By the above argument each G_n contains a countable subcollection G'_n covering space. Hence, space is completely separable and the theorem is established.

THE UNIVERSITY OF TEXAS

A REDUCED SET OF POSTULATES FOR ABSTRACT HILBERT SPACE*

BY A. E. TAYLOR

1. *Introduction.* An abstract Hilbert space is a normed linear space, or *vector space*, of infinite dimensionality, with a norm based on a Hermitian *inner product*, defined for all pairs of elements in the space. The space is, moreover, separable and complete according to this norm. The usual postulate system for Hilbert space, which was first stated abstractly by J. von Neumann, consists of five groups of postulates, or nineteen in all.†

The purpose of the present paper is to demonstrate the redundancy‡ of a number of the postulates, and to present a system of eleven independent postulates for a normed linear space with a Hermitian inner product. The adjunction of three more postulates, each of which is independent of the first eleven and the remaining two, then gives us a system which is equivalent to that of von Neumann, that is, it defines an abstract Hilbert space, and it is categorical.

A special feature of this postulate system is that the abstract relation called equality, and denoted, as usual, by the symbol $=$, enters on an equal footing with the operations defined in the space.§ Three of the eleven postulates are concerned with this

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† J. von Neumann, *Mathematische Grundlagen der Quantenmechanik*, 1932, pp. 19-24; M. H. Stone, *Linear Transformations in Hilbert Space*, Colloquium Publications of this Society, vol. 15, pp. 2-4.

‡ Some of these redundancies were noted simultaneously by a fellow-student, Mr. Ivar Highberg, and myself.

§ The postulational treatment of equality in vector spaces was suggested by A. D. Michal in a critique of postulate systems. See this Bulletin, vol. 39 (1933), Abstract No. 339.