SOME OBSERVATIONS ON THE THEORY OF FOURIER TRANSFORMS

BY E. HILLE, A. C. OFFORD, AND J. D. TAMARKIN[†]

1. *From a Letter by Offord.* On pages 770–771 of your paper,[‡] *On the theory of Fourier transforms*, you prove the following result.

LEMMA. *If $g(t)$ is in L_p, $1<p<\infty$, and if*

$$g(s,\ a) = \frac{1}{\pi}\int_{-\infty}^{\infty} g(t)\,\frac{\sin a(s-t)}{s-t}\,dt,$$

then

$$\lim_{u\to\infty}\int_{-\infty}^{\infty} |\, g(s) - g(s,\ a)\,|^{p} ds = 0.$$

You make no use of your hypothesis $p \leqq 2$ in this part of your paper. Now consider Berry's paper, *The Fourier transform identity theorem.*[§]

Write

$$G(s,\ a) = (2\pi)^{-1/2}\int_{-a}^{a} g(t)e^{-ist}dt,$$

and suppose $g(t)$ has a Fourier transform $G(s)$ in L_q, $1<q<\infty$, that is,

$$\lim_{a\to\infty}\int_{-\infty}^{\infty} |\, G(s,\ a) - G(s)\,|^{q} ds = 0.$$

THEOREM A (Berry). *If $g(s)$ has a Fourier transform $G(s)$ in L_q, $1<q<\infty$, and if $G(s)$ has a Fourier transform $\mathfrak{G}(s)$ in L_p, $1<p<\infty$, then $\mathfrak{G}(s) = g(-s)$.*

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† The present note contains excerpts from a letter by Offord to Tamarkin, and from a reply to this letter by Hille and Tamarkin. Before knowing the contents of Offord's letter Hille arrived independently at some of Offord's conclusions, as well as at extensions in other directions.

‡ E. Hille and J. D. Tamarkin, this Bulletin, vol. 39 (1933), pp. 768–774.

§ Annals of Mathematics, (2), vol. 32 (1931), pp. 227–232.