But it shows at any rate that any effort to find a contradiction between γ, δ, ϵ, or any combination of these, would be futile.

We have failed to find a map on which all of the operations of the impasse group are possible.

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ON THE EQUIVALENCE OF TWO METHODS OF DEFINING STIELTJES INTEGRALS*

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1. *Introduction.* The Stieltjes integral, $\int_a^b \phi(x)dg(x)$, was originally defined for $\phi(x)$ continuous on the closed interval $[a, b]$, and $g(x)$ of bounded variation. The limit which gives rise to this integral is taken as the length of the greatest sub-interval approaches zero. The above restrictions on $\phi(x)$ and $g(x)$, however, are not at all necessary for the existence of the limit, although it fails when the two functions have a common point of discontinuity. A generalization which permits such discontinuities is obtained by taking the limit in the sense of subdivisions,† to be defined below. The Riemann integral is an instance of the first type of limiting process, while the associated Darboux integrals are of the subdivision type. These can be shown to be of the first type as well. It is the purpose of this note to obtain general conditions for the equivalence of the two limits. By the introduction of the notion of interval functions a simple restriction on the integrand is found to be both necessary and sufficient.

2. *Subdivisions.* By a *subdivision*, σ, of the linear interval $X=[a, b]$ will be understood a finite set of adjacent sub-intervals whose sum is X. The *norm* of σ, the length of the greatest sub-interval, will be written N_σ. By the *product*, $\sigma' \cdot \sigma''$, of two subdivisions of X will be understood the subdivision which consists of all products of the form $I' \cdot I''$, where I' is an element of σ', and I'' is an element of σ''. It is assumed that every such

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† Moore and Smith, *A general theory of limits*, American Journal of Mathematics, vol. 44 (1922), pp. 102–121.