NOTE ON THE GEOMETRIC INTERPRETATION OF THE VANISHING OF A CERTAIN PROJECTIVE INVARIANT OF TWO CONICS*

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1. *Introduction.* The envelope of a line moving so that its pairs of intersections with two conics form a harmonic set is a third conic; the eight tangents to the given conics at their four points of intersection touch this envelope. The discriminant of this envelope is a relative invariant of weight four of the pair of ternary quadratic forms defining the given conic, and is the projective invariant mentioned in the title. When it vanishes, the envelope is degenerate, and the eight tangents mentioned above pass by fours through two points.†

Let the point-equations of the given conics be

$$\sum a_{ij}x_ix_j = 0, \quad a_{ij} = a_{ji}, \qquad \sum b_{ij}x_ix_j = 0, \quad b_{ij} = b_{ji}.$$

Then their line-equations are

$$\sum A_{ij}u_iu_j = 0, \qquad \sum B_{ij}u_iu_j = 0,$$

where A_{ij} is the cofactor of a_{ij} in the determinant $|a_{ij}|$, and similarly for B_{ij}. The line-equation of the envelope is then

$$\sum \alpha_{ij}u_iu_j = 0,$$

where α_{ij} is found by the Aronhold process:‡

$$\alpha_{ij} \equiv \sum \frac{\partial A_{ij}}{\partial a_{kl}} b_{kl} \equiv \sum a_{kl} \frac{\partial B_{ij}}{\partial b_{kl}}.$$

The condition that the envelope be degenerate is then $|\alpha_{ij}| = 0$. For want of a better term, we shall say that the two given conics are *related* if the envelope is degenerate. We shall proceed to prove a number of properties of related conics. Although most

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† Salmon, *Conic Sections*, 1917, pp. 306 ff., 345; Clebsch-Lindemann, *Vorlesungen über Geometrie*, 1932, vol. 1, pt. 1, pp. 513, 523.

‡ Clebsch-Lindemann, loc. cit., pp. 501, 513.