A NOTE ON THE EQUILIBRIUM POINT OF THE GREEN'S FUNCTION FOR AN ANNULUS

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1. *Introduction.* In a previous paper* the motion of the equilibrium point of the Green's function for a plane annular region was studied as the pole was shifted along a radius in the neighborhood of the geometric mean circle C_0.† The expression for dr/dr_0 on C_0, r being the distance of the equilibrium point from the center of the circles, r_0 that of the pole, is $-F_{r_0}/F_r$, where

$$F_{r_0} = \frac{\partial F}{\partial r_0} = -\frac{2}{R}\left[\frac{1}{2\log R} - \frac{1}{8} + \sum_{m=1}^{\infty}\frac{(-1)^m m}{R^m - 1}\right],$$

$$F_r = \frac{\partial F}{\partial r} = -\frac{2}{R}\left[\frac{1}{8} + \sum_{m=1}^{\infty}\frac{(-1)^m m}{R^m + 1}\right].$$

In these formulas 1 and R are the radii of the inner and outer circular boundaries of the region. It was shown by an application of a theorem of Schlömilch‡ that F_{r_0} does not vanish on C_0.

In this article this result and others are obtained by a method which seems better adapted to the problem.§

It is noticed that the function

$$f(z) = \frac{\pi}{\sin \pi z}\,\frac{z}{e^{az} - 1}, \qquad a = \log R,$$

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* D. M. Hickey, *The equilibrium point of Green's function for an annular region*, Annals of Mathematics, vol. 30 (1929), pp. 373–383.

† The Green's function for this region may be written in the form

$$g(M, M_0) = \log\frac{1}{MM_0} + \frac{1}{\log R}[\log R \log r_0 - \log r \log r_0/R]$$

$$- \sum_{m=1}^{\infty}\frac{1}{m}\,\frac{\cos m(\theta - \theta_0)}{R^{2m} - 1}\left\{r^m[r_0^m - r_0^{-m}] + r^{-m}\left[\left(\frac{R^2}{r_0}\right)^m - r_0^m\right]\right\}.$$

We take $F(r, r_0) = \partial g/\partial r$ for $r = r_0 = R^{1/2}$ and $\theta - \theta_0 = \pi$.

‡ *Über einige unendliche Reihen*, Zeitschrift für Mathematik und Physik, vol. 23 (1878), p. 132.

§ The suggestion that the method of contour integration and the theory of residues might prove useful was given by A. J. Maria.