A GENERALIZATION OF HARMONIC FUNCTIONALS*

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1. *Introduction*. In a recent paper W. V. D. Hodge† showed that most of the elementary properties of harmonic functions could be extended to harmonic functionals; on the other hand, we can extend the notion of harmonic functionals so that most of these elementary properties persist in this larger class. The present paper presents such an extension, and properties of the functionals it contains.

2. *Generalized Harmonic Forms*. Consider the $(p-1)$-form

$$\phi = A_{i_1\cdots i_{p-1}}dx^{i_1}\cdots dx^{i_{p-1}}, \quad (i = 1, \cdots, n), \tag{1}$$

in which the elements concerned obey the usual laws, with the exception that the dx's obey the non-commutative law of multiplication

$$dx^i dx^j = - dx^j dx^i. \tag{2}$$

Without loss of generality we assume that the summation in (1) is taken over all i for which $i_1 < \cdots < i_{p-1}$. If the A's have second partial derivatives which are continuous, then the form ϕ is said to be regular. The properties of such forms have been discussed by Cartan.‡

If the coefficients $A_{i_1\cdots i_{p-1}}$ are symbols, we shall speak of (1) as a symbolic form. In the present paper we shall be concerned only with symbolic linear forms:

$$\alpha = \alpha_i dx^i, \ \beta = \beta_i dx^i, \cdots.$$

The rules of combination for these are the same as for the forms of the type (1), except that the commutative law for the multiplication of a symbolic and a non-symbolic form does not hold.

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† W. V. D. Hodge, *A Dirichlet problem for harmonic functionals, with application to analytic varieties*, Proceedings of the London Mathematical Society, (2), vol. 36, part 4, pp. 257–303.

‡ E. Cartan, *Leçons sur les Invariants Intégraux*, 1922, Chapter 7.