THE ARITHMETIZED EXPANSIONS FOR CERTAIN DOUBLY PERIODIC FUNCTIONS OF THE THIRD KIND*

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1. *Introduction.* In this paper the explicit arithmetized Fourier developments are given for certain doubly periodic functions of the third kind, namely,

$$\Phi_{\alpha\beta\gamma}(x, y) = \vartheta_1'^3\vartheta_\alpha(x + y)/\vartheta_\beta^3(x)\vartheta_\gamma(y),$$
$$\Psi_{\alpha\beta\gamma}(x, y) = \vartheta_1'^3\vartheta_\alpha^2(x + y)/\vartheta_\beta^3(x)\vartheta_\gamma^2(y),$$

where x and y are independent complex variables, the ϑ's are the Jacobi theta functions, and α, β, γ are a certain sixteen triads out of the sixty-four which can be selected from the numbers 0, 1, 2, 3. The method of obtaining the expansions is essentially that used by Basoco† in obtaining the arithmetized developments for certain functions similar in form.

The applications to arithmetic lead to arithmetical paraphrase equations involving incomplete numerical functions in two variables. The complexity of the expansions is not adverse to simple and interesting arithmetical results, since simplicity can be gained by specialization while the complexity gives greater variety.

There are thirty-two expansions given in all. The domain of validity for x is $-I(\pi\tau)<I(x)<I(\pi\tau)$ in (1)–(8) and (17)–(24) inclusive, and $-(1/2)I(\pi\tau)<I(x)<(1/2)I(\pi\tau)$ for (9)–(16) and (25)–(32) inclusive; y is restricted in all expansions to $-(1/2)I(\pi\tau)<I(y)<(1/2)I(\pi\tau)$.

In all the series the outer summation refers to positive nonzero integers m, n, α in the exponent of q, and extends to all $m=1, 3, 5, \cdots$, to all $n=1, 2, 3, \cdots$, or $2, 4, 6, \cdots$, to all $\alpha=1, 5, 9, \cdots$. The coefficient of the power of q is in parentheses after the power. The inner summation is finite and refers to positive integral divisors t, τ, d, δ, of m, n, α, subject to further restrictions noted below. The integer τ is always odd. The

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† American Journal of Mathematics, vol. 54 (1932), p. 242.