

measure. Let L be a positive number and let $z(t, L)$ be equal either to $z_1(t)$ or $z_2(t)$ according as $[t/L]$ is even or odd. It is clear that $z(t, L)$ is an S.a.p. solution of $\Phi[z(t), F(t)] = 0$ for each positive L , and that all these solutions are essentially distinct. Of course many other solutions could be constructed in a similar way.

BROWN UNIVERSITY AND PRINCETON UNIVERSITY

ON A LEMMA OF FEJÉR*

BY LINCOLN LA PAZ

1. *Simple Integrals.* In an important paper L. Fejér† has verified and used the following lemma.

LEMMA A. *If for a problem of minimizing an integral*

$$(1) \quad I_1 = \int_{x_1}^{x_2} \phi(y') \cdot f(x, y) dx,$$

the Euler equation in normal form is

$$(1') \quad y'' = F(x, y, y'),$$

then for a problem of minimizing the integral

$$(2) \quad J_1 = \int_{x_1}^{x_2} [\phi(y')/f(x, y)] dx,$$

the Euler equation in normal form is

$$(2') \quad y'' = -F(x, y, y').$$

The following generalization of Fejér's lemma is proved in this note.‡

* Presented to the Society, December 2, 1933.

† L. Fejér, *Das Ostwaldsche Prinzip in der Mechanik*, *Mathematische Annalen*, vol. 61 (1905), p. 432.

‡ In everything that follows, the range of the indices i, j, k, μ, ν is from 1 to n and μ and ν are umbral.