A PROBLEM IN ARRANGEMENTS

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A problem in dynamics, recently considered by the author, gave rise to the following question.

Let us consider the n^r permutations of n different symbols $e_1, e_2, \cdots, e_n$ taken r at a time with repetitions allowed. *Can a sequence of these symbols be constructed such that each of these n^r permutations is found exactly once as a subsequence of r consecutive symbols in this sequence?*

Thus for $n=10$ and $r=5$ the question is equivalent to asking whether there exists a succession of digits such that every five-place number* is found exactly once among the consecutive digits of the succession. To consider a simpler example, in which the answer is readily given, let us take $n=3$ and $r=2$. Here the $n^r=3^2=9$ permutations are

$$e_1e_1,\ e_1e_2,\ e_1e_3,\ e_2e_1,\ e_2e_2,\ e_2e_3,\ e_3e_1,\ e_3e_2,\ e_3e_3,$$

and a sequence possessing the desired properties is

$$e_1e_3e_3e_2e_3e_1e_2e_2e_1e_1. \tag{1}$$

We now return to the general case and show that the answer to the question in italics above is in the affirmative for any pair of positive integers n and r whatsoever. The proof proceeds by first exhibiting an algorithm, the application of which is then shown to lead to a sequence of the symbols $e_1, e_2, \cdots, e_n$ possessing the desired properties.

From now on a subsequence of r consecutive symbols occurring in a sequence will always be designated as a section of r symbols. Thus $e_3e_2e_3e_1$ is a section of 4 symbols in the sequence (1).

The algorithm in question is built up out of the following three rules.

I. *Each of the first $r-1$ symbols is chosen equal to e_1.*

II. *The symbol a_m to be added to the sequence*

* By five-place numbers we mean, of course, those like 31342, 41231 etc., including those, however, such as 00123, 00005, etc.