

CONSTRUCTION OF A GROUPLESS AND HEADLESS TRIAD SYSTEM ON 31 ELEMENTS*

BY H. S. WHITE

No doubt the most interesting triad systems are those which admit a large group of substitutions, for example, those of the Heffter headless type,† admitting a cyclic substitution on all the elements. But on a sufficiently large set of elements systems can be constructed which have smaller groups or even no group.‡ The structure of such systems is worth recording for the same reason as, in the theory of algebraic plane curves, not only those with no singular points, but also those with all admissible singularities and actual aggregates of singularities demand enumeration.

The least number of elements in a groupless system is fifteen.§ For thirty-one elements, Reiss's method can be adapted to yield a triad system with no group,|| but a system built by that method has necessarily a "head", namely a triad (sub-)system on fifteen, a Δ_{15} , which itself has no group. This class of Δ_{31} 's, millions in number, were at their first appearance a surprise, since groupless systems previously known were also headless. But the obvious question was still unanswered: is there any Δ_{31} which is both groupless and headless? I shall here exhibit one such by actual construction, and describe the application of the method of sequences and indices by which it is proved to be both groupless and headless. This method is of course tedious, arousing the wish that some clearer insight and more sweeping form of argument might be attained.

* Presented to the Society, September 11, 1930.

† L. Heffter, *Ueber Tripelsysteme*, *Mathematische Annalen*, vol. 49 (1897), p. 101.

‡ H. S. White, *The multitude of triad systems on thirty-one elements*, *Transactions of this Society*, vol. 16 (1915), pp. 13-19.

§ See reference to DePasquale and Brunel in article by F. N. Cole, *The triad systems of thirteen elements* *Transactions of this Society*, vol. 16 (1913), p. 1.

|| H. S. White, *loc cit*