and m/n lies between constants which depend on δ. Then it is quite easy to prove that

$$|\sigma_n^*(\theta)| = |n\sigma_n(\theta) - m\sigma_m(\theta)|/(n-m) \leqq MB,$$

while your argument shows at once that $|R_n(\theta)| \leqq 2B + \vartheta A$, where ϑ is small by choice of $m/n \cdots$."†

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NOTE ON THE FORM OF A FIRST-ORDER PARTIAL DIFFERENTIAL EQUATION‡

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In this paper we give a simple proof of the fact that the non-singular solutions of a first-order partial differential equation can be obtained by equating to zero solutions of an associated equation in which the dependent variable does not appear explicitly. The usual proof§ of this property makes extensive use of the *complete integral*, and to be given rigorously would require considerations at one stage nearly as involved as our entire proof.|| Our proof has no reference to complete integrals. The results, as usual, hold in the small. Interest in this question arises from the treatments of equations in which the unknown does not appear explicitly.

THEOREM. *Let* $f(x_1, \cdots, x_n, z, p_1, \cdots, p_n) = f(x, z, p)$ *be of class* C''¶ *in a neighborhood of an initial element* (a, b, p^0) *for which* $f=0$ *and* $f_{p_1} \neq 0$. *Let*

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† Professor Fekete, to whom I communicated this letter of Paley in September 1933, has worked out completely the proof sketched by Paley. Moreover, Fekete generalized considerably Paley's theorem and extended it also to the trigonometric series of H. Bohr.

‡ Presented to the Society, March 31, 1934.

§ See, for example, E. Goursat, *Équations aux Dérivées Partielles*, 1921, pp. 48–49 and 159.

|| A complete integral yielding elements at a given point does not necessarily provide any given integral element at the point.

¶ A function of class $C^{(k)}$ is one having continuous kth partial derivatives.