The possibility of setting $p_k=0$, $k>0$, for large values of t will depend on whether $\phi(t)$ admits an asymptotic expansion along the positive real axis. The region of definition of $\phi(t)$ as well as the possibility of its expansion by the transformation (11) may be inferred from general theorems in the theory of divergent series*. With due regard to the distribution of the singularities of $f(p)$, (6) and (11) may be used when $f(p)$ is expanded in a two-way Laurent series.

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SYMMETRIC, SELF-DUAL, RATIONAL PLANE CURVES OF ODD ORDER†

BY D. C. DUNCAN

The writer has established the existence‡ of the real, non-degenerate, completely symmetric, self-dual, *elliptic* curves of order $4k+2$, and has indicated an extremely simple plan of sketching the loci approximately. In pursuing the program there indicated, we now establish the existence of completely symmetric *rational* self-dual curves of order $2k+1$, and indicate the corresponding easy mode of sketching them approximately. These loci also are invariant under $4k+2$ collineations and $4k+2$ correlations of which $2k+1$ are polarities by real rectangular hyperbolas and one a polarity by an imaginary circle. Moreover, the $(2k+1)^2$ foci have an interesting distribution and admit rather easy determination. In conclusion, the equations in Cartesian coordinates of the curves of the lowest three orders, 5, 7, and 9, are listed, together with a sketch of the 9-ic depicting all singular elements, real foci, and real autopolarizing conics. In a subsequent paper the elliptic and rational self-dual completely symmetric curves of orders $4k$, $2k+1$, and $4k$, $4k+2$, respectively, are to be exhibited, therewith terminating the present discussion.

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* See, for example, Borel et Bouligand, *Leçons sur les Séries Divergents*, Gauthier-Villars, 1928, Chapter 4.

† Presented to the Society under a slightly different title, December 2, 1933.

‡ This Bulletin, vol. 39 (1933), pp. 809–813