# A NOTE ON NILPOTENT ALGEBRAS IN FOUR UNITS*

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1. *Introduction.* In volume 9 of the Transactions of this Society R. B. Allen gave, without proof, a classification of all associative nilpotent algebras in $n \leqq 4$ units.† These results were later verified by A. A. Albert for $n \leqq 3$.‡

I have recently completed a reclassification of nilpotent algebras in four units and have discovered several serious errors in Allen's results. Although he stated that his classification separated the algebras into classes of non-equivalent and non-reciprocal algebras, he actually did not accomplish this, as he gave several superfluous cases. Moreover, he erroneously listed certain classes of non-associative algebras which must be replaced by similar classes of associative algebras.

In §2 I shall prove the validity of the following revision of Allen's 16 classes (labelled A) into my §9 classes (labelled G). Allen's classes of *irreducible algebras* are

(1A) $e_1^2 = ae_4, \quad e_2e_3 = be_4 = e_3^2, \quad e_3e_1 = e_4 = -e_3e_2,$

(2A) $ae_1^2 = e_2e_1 = e_2^2 = e_3e_2 = be_3^2 = e_4,$

(3A) $e_1^2 = e_3e_2 = e_4,$

(4A) $ae_1^2 = e_2e_1 = e_2^2 = be_3^2 = e_4,$

(5A) $ae_1^2 = e_2^2 = be_3^2 = e_4,$

(6A) $e_1^2 = e_3, \quad e_2e_1 = e_4, \quad e_1e_2 = e_3 + ae_4, \quad e_2^2 = be_3 + ce_4,$

(7A) $e_1^2 = e_3, \quad e_2e_1 = e_4, \quad e_2^2 = ae_3 + e_4,$

(8A) $e_1^2 = e_3, \quad ae_2^2 = e_2e_1 = e_4,$

(9A) $e_1^2 = e_3, \quad e_2^2 = e_4,$

(10A) $e_1^2 = e_3, \quad e_1e_2 = ae_2e_1 = e_2^2 = e_4,$

(11A) $e_1^2 = e_3, \quad e_1e_2 = ae_2e_1 = e_4,$

(12A) $e_1^2 = ae_2^2 = e_3, \quad e_2e_1 = ae_1e_2 = e_4,$

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† Transactions of this Society, vol. 9, pp. 213.

‡ In his master thesis, pp. 5–7.

§ As may be observed, my own classes are but minor revisions of Allen's classes. These revisions are necessary in order that all algebras may be included.