A NOTE ON THE JACOBI CONDITION FOR PARAMETRIC PROBLEMS IN THE CALCULUS OF VARIATIONS*

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1. *Introduction.* An elegant treatment of the Jacobi condition for parametric problems in $(y_1 \cdots y_n)$-space has been given by Bliss.† He defines conjugate points in terms of solutions η_i of the Jacobi equations which satisfy the relations $y_i'\eta_i = 0$. With the help of these solutions he establishes criteria for conjugate points in terms of the general solutions of the Jacobi equations. Since there are but $2n-2$ linearly independent solutions η_i satisfying the conditions $y_i'\eta_i = 0$, the treatment given by Bliss is quite different from that usually given for non-parametric problems. It is well known that the methods of Bliss are still applicable if the equation $y_i'\eta_i = 0$ is replaced by an equation such as $y_i'\eta_i' = 0$, $y_i'\eta_i' = \text{constant}$, and others.‡ In the present paper we use the equation $y_i'\eta_i' = \text{constant}$ in defining conjugate points and obtain the same results as Bliss. The method used is, however, quite different from that of Bliss and has the advantage that it is almost identical with that usually given for the non-parametric problems. This follows because there are $2n$ linearly independent solutions of the type considered in this paper. Other equations for which the method here used is still applicable are also discussed.

2. *The Necessary Condition of Jacobi.* The problem to be considered is that of minimizing an integral

$$I = \int_{t_1}^{t_2} f(y_1, \cdots, y_n, y_1', \cdots, y_n')dt = \int_{t_1}^{t_2} f(y, y')dt$$

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† Bliss, *Jacobi's condition for problems of the calculus of variations in parametric form*, Transactions of this Society, vol. 17 (1916), pp. 195–206.

‡ Graves, *Discontinuous solutions in space problems of the calculus of variations*, American Journal of Mathematics, vol. 52 (1930), p. 17. See also Wren, *A new theory of parametric problems in the calculus of variations*, Contributions to the Calculus of Variations, 1930, The University of Chicago Press, pp. 175–185.