THE PROBABILITY LAW FOR THE SUM OF n INDEPENDENT VARIABLES, EACH SUBJECT TO THE LAW $(1/(2h))\text{sech}(\pi x/(2h))$*

BY W. D. BATEN

1. *Introduction.* Let the probability of selecting the chance real variable x from the interval $(x, x+dx)$ be to within infinitesimals of higher order, the quantity $(1/(2h))$ sech $(\pi x/(2h))dx$. This hyperbolic secant probability or frequency function has been used by others. Roa considered this function in many details as a generating function for frequency functions and gave numerical tables pertaining to it.† Fisher obtained as a special case a type of this frequency law for the frequency of the "intraclass" correlation coefficient.‡ Dodd investigated this probability function as a particular case when considering measurements under general laws of errors.§ The author obtained the law for the sum of n independent variables when each is subject to this hyperbolic law but was not able to express the sum function without the use of an integral.‖

The object of this article is to find the probability function for the sum $\sum_{i=1}^{n} x_i$ when each variable x_i is subject to the probability function $(1/(2h))$ sech $(\pi x_i/(2h))$, or to find the probability to within infinitesimals of higher order that

$$u \leqq \sum_{i=1}^{n} x_i \leqq u + du.$$

2. *Case* I: n *Finite.* If a general method due to Dodd¶ be applied to this hyperbolic secant law, the probability law for the sum of n independent variables is

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† E. Roa, *A number of new generating functions with applications to statistics*, Thesis, University of Michigan, 1924.

‡ R. A. Fisher, *On the probable error of a coefficient of correlation deduced from a small sample*, Metron, vol. 1 (1920–21), pp. 3–32.

§ E. L. Dodd, *Functions of measurements under general laws of errors*, Skandinavisk Aktuarietidskrift, 1922, No. 3, pp. 134–158.

‖ W. D. Baten, *Frequency laws for the sum of n variables which are subject to given frequency laws*, Metron, vol. 10 (1932), No. 3, pp. 75–91.

¶ E. L. Dodd, *The frequency law of a function of variables with given frequency laws*, Annals of Mathematics, (2), vol. 27 (1925–26), p. 13.