THE CALIFORNIA COLLOQUIUM

Differential Equations from the Algebraic Standpoint. By J. F. Ritt. New York, American Mathematical Society, 1932. x+172 pp.

The author sets himself the task of developing a theory of elimination which will reduce the existence problem for a finite or infinite system of algebraic differential equations to the application of the implicit function theorem taken with Cauchy's theorem in the ordinary case and Riquier's in the partial. The number of unknowns and independent variables is considered finite and fixed at the outset. The left members of the system are polynomials in the unknowns and their derivatives. The number of derivatives entering a given polynomial, presumably, is finite, though the number varies from polynomial to polynomial and may increase without limit in a sequence of polynomials.

The quest being, essentially, for holomorphic solutions of systems with holomorphic coefficients, every solution must satisfy what Riquier has named the *prolonged system*, which comprises all equations arising by differentiating the given equations. The author calls an algebraic differential system reducible or irreducible according as its prolonged system is algebraically reducible or not. One of the main currents of thought in the book seems to spring from the remark that an algebraically irreducible system may be reducible when regarded as a differential system.

The course followed by the author in accomplishing his purpose will now be indicated in summary fashion. It is shown that every system Σ is equivalent to a finite number of closed irreducible systems

$$\Sigma_1, \Sigma_2, \cdots, \Sigma_s \tag{1}$$

in the sense that every solution of Σ satisfies at least one Σ_i and every solution of a Σ_i satisfies Σ. For the case of a finite system, a theoretical process is developed (§67) for obtaining each of these irreducible systems in terms of a corresponding *basic set* B_i, which contains at most one equation corresponding to each unknown and in addition has other special defining properties. A *normal solution* of B_i is one for which none of a certain associated set of polynomials, called the *separants*, vanishes identically. Every solution of Σ_i satisfies the corresponding B_i, and, in the nature of a partial converse, it is proved that every normal solution of B_i satisfies the corresponding Σ_i (§66). Furthermore, every solution of Σ_i which is not normal for B_i satisfies a system Σ_i which has a basic set B_{ij} of lower rank than B_i. This makes it possible to replace (1) by a like finite set having the additional property that every solution of the original system Σ is a normal solution for at least one of the basic sets which have been associated with the Σ_i.

The above, which applies literally only to the ordinary case, also describes the author's treatment of the partial case, if normal solution is replaced by regular, the latter type being one for which the set of non-vanishing polynomials has been augmented by the corresponding *initials*.

The final step is the development of a theoretical process for finding a finite set of equations equivalent to a closed irreducible system Σ_i. This is accom-