

RADÓ ON PLATEAU'S PROBLEM

On the Problem of Plateau. By Tibor Radó. *Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 2, Heft 2.* Berlin, Springer, 1933. 109 pp.

This report is an interesting and informative account of the recent work on the problem of Plateau—the problem which requires a minimal surface, or surface of least area, bounded by a given closed curve—dealing especially with the contributions made in recent years by Garnier, Douglas, Radó, and McShane. The survey consists of six chapters.

Chapter 1 reviews some general notions concerning length and area, with the implications of the various definitions that have been proposed for the latter. The definition of area most significant for the work on the Plateau problem is that given by Lebesgue in his thesis: the area of a surface S is the *least* limit towards which the area of a polyhedral surface P can tend when P approaches to S . This definition is associated with the essential lower semi-continuity property of area.

The importance for the problem of Plateau of monotonic transformations of the unit circumference C into the given contour Γ is pointed out, that is, the totality of ways of representing Γ as a topological image of C . The introduction of these monotonic transformations into the problem, as well as the important remark of the *compactness* of the totality of them, is due to the present reviewer in his abstracts in this Bulletin and his European lectures of 1929. This property of compactness makes it possible to prove the attainment of the minimum of any lower semi-continuous functional having these monotonic transformations as range of the argument, by a simple and classical procedure going back to Weierstrass and Fréchet. The totality of surfaces S bounded by a given contour does not form a compact set; it was the necessity of obtaining such a set as range of the argument of the functional, area of S , that underlay the restrictions to which the contour was subjected in the earlier work, notably that of Lebesgue and Haar.

In Chapter 2 are reviewed some facts from the differential geometry of minimal surfaces, necessary in the sequel, especially the formulas of Monge, Weierstrass, and Schwarz.

Chapter 3 begins with five modes of formulation P_1, \dots, P_5 of the Plateau problem, varying somewhat in their implications. In these formulations a minimal surface is taken to be one defined by the Weierstrass formulas

$$x_i = \Re \phi_i(w), \quad \sum_{i=1}^3 \phi_i'^2(w) = 0,$$

which amount to the condition of zero mean curvature for the surface together with the requirement of its conformal mapping on a circular disc.

If we require a surface which not only is minimal in the sense of the preceding formulas, but also has least area, then we have what Radó calls the "simultaneous problem." Examples, due to Schwarz, are cited of minimal surfaces which do not have the least area property in their entire extent.