THE APPARENT CONTOUR OF THE GENERAL V_3^n IN S_4*

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It is the purpose of this paper to study the surface obtained by intersecting with a 3-space the hypercone of tangents drawn to a hypersurface V in S_4 from a point O. Or we may say that we project from O upon S_3 the surface which is the intersection of V and the first polar hypersurface of O. It may be recalled that when V is the locus of lines that meet four planes, and therefore a fifth, and O is a general point of V, the Kummer surface is the result. In this paper V will be assumed non-singular and of order n. But the surface obtained has all the ordinary singularities, including a cuspidal curve. The two cases when the center of projection is not a point of V, and when it is on V, will be considered in that order.

If V is a non-singular hypersurface in S_4 of order n, and O any point not on V, the hypercone of tangents to V from O meets a 3-space in a surface F' of order $n(n-1)$. For a line in S_3 determines with O a plane that meets V in a curve to which $n(n-1)$ tangents can be drawn from O. As remarked, F' is the projection from O of the surface F, which is the intersection of V and the first polar hypersurface of O. The second polar hypersurface of O meets F in a curve C of order $n(n-1)(n-2)$ whose projection is the cuspidal curve of F'. For, if P is a general point of C, the tangent plane to F at P passes through O. A 3-space containing OP, but not the tangent plane, meets F in a curve to which OP is a simple tangent. Hence its projection is a plane curve on F', having a cusp at P', the projection of P. The cuspidal tangent is the intersection with S_3 of the osculating plane to the space curve at P. As the 3-space section through OP varies there are ∞^1 distinct osculating planes. They lie in the 3-space which is tangent to V at P. For this 3-space intersects V in a surface having a node at P, and intersects the first polar of O in a surface whose tangent plane at P is the tangent plane to F. The second of these surfaces is the polar of O with respect to the first. Furthermore, OP meets V in three points at P. From all of which

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