

ear dimensions. We say that, for two spaces of type (F) , (1) $\dim_l E \leq \dim_l E_1$ if E is isomorphic with a linear closed sub-space of E_1 ; that $\dim_l E = \dim_l E_1$ if also (2) $\dim_l E_1 \leq \dim_l E$, while $\dim_l E < \dim_l E_1$ if (1) is satisfied but (2) is not. If neither (1) nor (2) is satisfied, the linear dimensions of E and E_1 are said to be incomparable. It should be observed that, while two isomorphic spaces are of the same linear dimensions, there exist separable spaces (B) which are not isomorphic although their linear dimensions are the same. Special attention is given here to spaces (L^p) and (l^p) . It is shown that if $\dim_l(L^p) = \dim_l(l^q)$, where $p, q > 1$, then $p = q$; if $1 < p < 2 < q$, then the linear dimensions of (L^p) and (l^q) are incomparable; if $1 < p \neq 2$, then $\dim_l(L^2) < \dim_l(L^p)$; if $\dim_l(L^p) < \dim_l(L^q)$, where $p, q > 1$, then $p = q = 2$; the condition $p = q = 2$ is necessary and sufficient that $\dim_l(L^p) = \dim_l(l^q)$; if $1 < p \neq 2$, then $\dim_l(L^p) > \dim_l(l^p)$. The Appendix (18 pp.) contains an additional discussion of the weak convergence of elements and of functionals. The book closes with 21 pages of additional remarks containing historical information and statements of various generalizations of the results in the text, as well as numerous unsolved problems. These remarks reveal that the whole subject is in a state of vigorous development. The usefulness of many of these remarks would be considerably greater if they contained hints indicating how the results in question have been obtained. The exposition as a rule is clear and detailed, although not easy in several places. The number of misprints (of which only two are corrected in the list of Errata) is not negligible.

In conclusion the reviewer may express his conviction that Banach's monograph should occupy a permanent and honorable place on the desk of every one who is interested in the theory of linear operations, to be replaced only by subsequent, corrected and augmented editions, which undoubtedly will follow before long.

J. D. TAMARKIN

SAKS ON INTEGRATION

Théorie de l'Intégrale. By Stanislaw Saks. Warsaw, Monografje Matematyczne, Vol. II, 1933. vii+290 pp.

The present volume is the second one of the series *Monografje Matematyczne*. It is a translation, entirely revised and augmented by several important chapters, of the author's Polish book, *Zarys Teorii Ciągi*, Warsaw, 1930. It fills in a serious gap in the literature of the real function theory, and of the theory of differentiation and integration, which has been acutely felt during all the recent period of vigorous growth and development of these disciplines. While Hermite, together with a large part of his contemporaries and immediate successors, including Poincaré, contemplated with horror the pathological cases of functions without derivatives, the ideas and methods created precisely for handling such bad cases turned out to be extremely fruitful and indispensable for the treatment of most classical and venerated problems of analysis. Far from bringing in the feared anarchy and disorder, they have allowed us in many instances to reach a harmony and completeness of results which were entirely out of the reach of older classical methods. The problems