A NEIGHBORHOOD TREATMENT OF GENERAL TOPOLOGICAL SPACES*

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In this paper we deal with all the subsets of a space R of elements called points. Each point p of R may have associated with it certain subsets of R called *neighborhoods of* p so that it is determined by some rule whether or not any particular set is a neighborhood of any particular point. In the most general case no assumptions are made such as that every point has at least one neighborhood associated with it, or that the point p is an element of the neighborhood associated with it.

The purpose of this paper will be to consider (1) the various ways of defining limit point in terms of neighborhoods; (2) what properties must be assumed concerning the neighborhoods in order that limit point have certain well known properties. We shall start with the following new definition of limit point which, although somewhat peculiar in character, is found to be most convenient for the case of the general topological space.†

DEFINITION A. A point p is said to be a *limit point* of a set E if every neighborhood of p that contains $C(E)$‡ contains at least one point of E.

DEFINITION B. A point p is said to be *interior* to the set E if it is not a limit point of $C(E)$.

THEOREM 1. *The set N is a neighborhood of the point p if and only if p is interior to N.*

PROOF. (1) Suppose p were not interior to N; then p would be a limit point of $C(N)$ and hence every neighborhood of p that contains the complement of $C(N)$ must contain a point of $C(N)$. Now N is a neighborhood of p by hypothesis, but it contains no point of $C(N)$, which gives us a contradiction. (2) If p is not a limit point of $C(N)$, then there is a neighborhood of p which contains N but no point of $C(N)$, hence N is the neighborhood.

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† M. Fréchet, *Les Espaces Abstraites*, p. 166; E. W. Chittenden, Transactions of this Society, vol. 31 (1929), pp. 290–321.

‡ The symbol $C(E)$ means the complement of the set E. The whole definition implies that p is a limit point of E if $C(E)$ is not a neighborhood of p.