ON THE NUMBER OF $(q+1)$-SECANT S_{q-1}'S OF A CERTAIN V_k^n IN AN $S_{qk+q+k-1}$

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In this note we are concerned only with those k-dimensional non-developable varieties which are rational loci each of ∞^1 $(k-1)$-spaces. By a rational locus of ∞^1 $(k-1)$-spaces we mean one whose $(k-1)$-spaces can be put in a one-to-one correspondence with the points of a straight line. Let such a locus or variety, V_k^n, of order n be given in an S_r. Now in S_r there are $\infty^{q(r-q+1)}$ $(q-1)$-spaces. For a $(q-1)$-space to meet V_k^n $q+1$ times is equivalent to $(q+1)(r-q-k+1)$ simple conditions. In order that the number, N, of $(q-1)$-spaces $(q+1)$-secant to V_k^n, that is, having $q+1$ points of simple incidence with V_k^n, be finite, we must have $(q+1)(r-q-k+1)=q(r-q+1)$ or $r=qk+q+k-1$. It is our purpose to determine this number N of $(q+1)$-secant S_{q-1}'s of V_k^n in $S_{qk+q+k-1}$.

For this purpose we find it convenient to consider the V_k^n in question as the projection of a $V_k'^n$ in a higher space $S_{r'}$. This $V_k'^n$ may always be regarded as the locus of ∞^1 $(k-1)$-spaces joining corresponding points of k rational, projectively related curves $C^{n_1}, C^{n_2}, \cdots, C^{n_k}$ of respective orders $n_1, n_2, \cdots, n_k$, where $n_1+n_2+\cdots+n_k=n$. The $S_{r'}$ containing $V_k'^n$ must be such that $r'\leqq n+k-1$. If $r'=n+k-1$, $V_k'^n$ is said to be normal in S_{n+k-1}. It is only necessary to consider this normal $V_k'^n$.

Let the k curves be given parametrically by

$$C^{n_1}\quad x_0:x_1:\cdots:x_{n_1}=t^{n_1}:t^{n_1-1}:\cdots:1,$$
$$x_{n_1+1}=x_{n_1+2}=\cdots=x_{n+k-1}=0;$$
$$C^{n_2}\quad x_0=x_1=\cdots=x_{n_1}=0,$$
$$x_{n_1+1}:x_{n_1+2}:\cdots:x_{n_1+n_2+1}=t^{n_2}:t^{n_2-1}:\cdots:1,$$
$$x_{n_1+n_2+2}=x_{n_1+n_2+3}=\cdots=x_{n+k-1}=0;$$

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$$C^{n_k}\quad x_0=x_1=\cdots=x_{n-n_k+k-2}=0,$$
$$x_{n-n_k+k-1}:x_{n-n_k+k}:\cdots:x_{n+k-1}=t^{n_k}:t^{n_k-1}:\cdots:1.$$