ON THE LEXIS THEORY AND THE ANALYSIS OF VARIANCE*

BY H. L. RIETZ

In a paper† published in 1921 involving a generalization of the Lexis theory for the classification of statistical series with regard to their dispersion into Bernoulli, Lexis, and Poisson series, Coolidge based much of his reasoning on the following fundamental dispersion theorem.

If n independent quantities $y_1, y_2, \cdots, y_n$ be given, their expected values being $a_1, a_2, \cdots, a_n$, while the expected values of their squares are $A_1, A_2, \cdots, A_n$, respectively, and if we agree to set $y=(1/n)\sum_{i=1}^{n} y_i$, $a=(1/n)\sum_{i=1}^{n} a_i$, then the expected value of the variance $(1/n)\sum_{i=1}^{n}(y_i-y)^2$ is

$$(1)\qquad \frac{1}{n}\left[\frac{n-1}{n}\sum_{i=1}^{n}(A_i - a_i^2) + \sum_{i=1}^{n}(a_i - a)^2\right].$$

In setting up criteria for the practical classification of actual statistical series, Coolidge followed the customary procedure of introducing approximations by replacing $(n-1)/n$ by 1.

By avoiding this approximation in the present paper but otherwise proceeding along the lines followed by Coolidge, we shall arrive at certain important results of R. A. Fisher in his analysis of variance. The different estimates of variance used by R. A. Fisher seem to have been obtained largely by inferences based on the number of degrees of freedom of the variates rather than upon formal mathematical proofs. In fact, the intuitional element is so prominent in certain of these inferences based on the number of degrees of freedom that mathematicians rather generally hesitate to accept the results as mathematically established, although there is much general evidence in favor of the correctness of the conclusions. For this reason, it seems of interest to show how certain of the results in question can be derived by formal developments that follow closely the reasoning of Coolidge in his generalization of the Lexis theory.

---

* Presented to the Society, December 28, 1931.

† This Bulletin, vol. 27 (1921), p. 439.