ON CERTAIN CHARACTERISTICS OF k-DIMENSIONAL VARIETIES IN r-SPACE

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An algebraic variety of k dimensions in r-space has numerous characteristics besides its order n. The characteristics of algebraic curves and surfaces and the relations they satisfy are known. In this paper we consider a variety V_k of dimension k greater than 2. Assuming it to be the complete intersection of $r-k$ hypersurfaces of orders $n_1, n_2, \cdots, n_{r-k}$ respectively in S_r, we derive the formulas for a few of its characteristics in terms of the n's and incidentally obtain the relations connecting them. To avoid unnecessary length of discussion we consider somewhat in detail the V_3 in S_7 only and then give the results without demonstration for V_k in S_r.* The method here employed is the familiar one of complete degeneration which we have repeatedly made use of elsewhere in dealing with problems of similar nature.†

Now for the purpose of enumerating the characteristics of V_k and obtaining their relations we may regard the variety as belonging to an S_{2k+1}, for a V_k belonging to an S_r where $r>2k+1$ possesses no characteristics not possessed by a V_k of S_{2k+1}. If we project V_k from a general S_{t-1} of S_{2k+1} on to an S_{2k+1-t} $[0\leqq t\leqq k]$ of S_{2k+1}, we have for projection a $V_k^{(t)}$ possessing a double $(t-1)$-dimensional variety $V_{t-1}^{b_{t-1}}$ of order b_{t-1} and a pinch $(t-2)$-dimensional variety $V_{t-2}^{j_{t-2}}$ of order j_{t-2} lying on $V_{t-1}^{b_{t-1}}$. From a general point of S_{2k+1-t} we can construct ∞^t lines forming a $(t+1)$-dimensional cone of order b_t each meeting $V_k{}^{(t)}$ in two distinct points. We say that $V_k^{(t)}$ has an apparent double $V_t^{b_t}$ of order b_t. Again, from a general point of S_{2k+1-t} a t-dimensional cone of ∞^{t-1} lines of order j_{t-1} can be constructed tangent to

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* Some work has been done along this line. See C. Segre, *Mehrdimensionale Räume*, Encyklopädie der Mathematischen Wissenschaften, III$_2$, 7, pp. 922–927.

† B. C. Wong, *On the number of apparent multiple points of varieties in hyperspace*, this Bulletin, vol. 36, pp. 102–106; and *On surfaces in spaces of four and five dimensions*, this Bulletin, vol. 36, pp. 861–866.