THREE THEOREMS APPLICABLE TO VIBRATION THEORY*

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1. *Introduction.* In discussions of the theory of free vibrations it is sometimes stated without proof that "the fact that the period of vibration is independent of the amplitude shows that the restoring force is linear." The present paper includes a proof of this statement and a brief consideration of the case of a non-linear restoring force.

2. *Hypotheses and Existence of Period Integral.* (A) Let the restoring force $f(x)$ be a continuous function of the distance x from the position of equilibrium on the interval

$$0 \leqq x \leqq D. \tag{1}$$

Let $f'(x)$ exist on this interval. Let $f(0) = 0$ and

$$f(x) > 0,\ f'(x) > 0 \text{ on } 0 < x \leqq D.$$

We suppose also that $f(x)$ is symmetric in $x = 0$; that is, we define the function over the interval $-D \leqq x \leqq 0$ so that

$$f(+x) = -f(-x).$$

(B) Take the derivative

$$f'(0) = b > 0.$$

(C) Let the unilateral derivative $f''(0^+)$ exist.

The differential equation of vibration is

$$\frac{d^2x}{dt^2} = -f(x),$$

and one obtains as an integral

$$t = \int_0^x (C - 2F(x))^{-1/2}dx + \text{const.},$$

where $F(x) = \int_0^x f(x)dx$ and C is a constant of integration. Let a

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