# AN INVERSIVE ALGORITHM*

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Viggo Brun‡ has given an algorithm for calculating directly the $n$th prime number from certain values of the function $\pi(x)$, the number of primes $\leqq x$. It is the purpose of this note to show that this algorithm is not peculiar to primes, but on the contrary, may be used to exhibit the $n$th member of any infinite class $C$ of positive integers. With this degree of generality it is possible to use the algorithm to obtain identities between numerical functions.

The algorithm may be described as follows. Associated with the class $C$ is the enumerative function $\theta(x)$ giving the number of members of $C$ which are $\leqq x$. If $n$ is any positive integer we form the sequence

$$n_0, n_1, n_2, \cdots, n_r, \cdots, \tag{1}$$

whose terms are defined as follows:

$$\begin{aligned} n_0 &= n, \\ n_1 &= n - \theta(n_0), \\ n_2 &= n - \theta(n_0 + n_1), \\ n_3 &= n - \theta(n_0 + n_1 + n_2), \\ &\cdots\cdots\cdots\cdots \\ n_r &= n - \theta(s_r), \end{aligned} \tag{2}$$

where, for brevity, we have written

$$s_r = n_0 + n_1 + n_2 + \cdots + n_{r-1}.$$

We have then the following theorem.

THEOREM. *The terms of the sequence* (1) *do not increase, and ultimately become and remain zero. If $k$ is the rank of the first zero term of* (1), *then $s_k$ is the nth member of the class $C$.*

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‡ Kongelige Norske Videnskaps-Selskabet, vol. 4, pp. 66–69.