

Gewöhnliche Differentialgleichungen. By G. Hoheisel. Zweite, verbesserte Auflage. Sammlung Götschen. Berlin and Leipzig, Walter de Gruyter, 1930. 159 pp.

The first edition of this little book appeared in 1926 and was reviewed in this Bulletin, volume 35, page 136, where it was remarked (by another reviewer) that "as a textbook for the American or English student, it will not be found entirely satisfactory." The improvement in the second edition is not noticeable and the book is still unsuitable as a text, though it may serve satisfactorily as collateral reading.

There are no problems and the examples chosen to illustrate the theory are always of the simplest type, giving sometimes only a vague impression of the situation. It seems particularly unfortunate and misleading to illustrate a singular solution by the example $(y' - xy)(y' - y^2) = 0$. The usual assumption that the equation $F(x, y, y') = 0$ is irreducible is not made and the example gives an impression of a quibbling triviality.

The book covers a surprising amount of ground and in some places considerable skill is shown in the introduction and brief, though satisfactory, treatment of a topic. In other places, however, considerable carelessness appears, as in the treatment of singular points of linear differential equations on page 101. The equation is taken first in the form

$$p_0(x)y'' - p_1(x)y' - p_2(x)y = q(x),$$

where $p_0, p_1, p_2,$ and q are convergent series in integral powers of x . The point $x=0$ is a singular point if $p_0(0) = 0$. Assuming, for simplicity, that $q(x) \equiv 0$ and that $x=0$ is a simple zero of p_0 , so that $p_0(x) = x\pi_0(x)$ where $\pi_0(0) \neq 0$, the equation is divided by $\pi_0(x)$ and put in the form $xy'' - \pi_1(x)y' - \pi_2(x)y = 0$. This equation is then multiplied by x and written $x^2y'' - xh_1(x)y' - h_2(x)y = 0$, which is the form most convenient for the investigation of regular singular points.

The author does not mention that this is a highly specialized case because $h_2(x)$ must contain x as a factor. The result of the specialization is that one of the two exponents of the equation for the point $x=0$ always vanishes. Ignoring this fact, the treatment proceeds as in the general case including a discussion of conjugate complex roots of the indicial equation, although the only differential equation under consideration has as indicial equation a quadratic with one zero root. Incidentally the terminology "indicial equation" and "exponents" is not used.

While there is no statement in this section that can be called incorrect, the impression is clearly given that a simple zero of p_0 is a singular point of the differential equation leading to the general situation discussed later. The essential fact that p_0 may carry x^2 , but no higher power, as a factor is not mentioned. The reader approaching this topic for the first time is not likely to get a correct understanding of it.

The desire to simplify the expositions of each topic as much as possible produced unfortunate results in the case of the singular points mentioned above, but in other places the same method proves very effective and the careful reader may find the book quite helpful in connection with a first study of the subject.

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