

DICKSON ON THEORY OF NUMBERS

Studies in the Theory of Numbers. By L. E. Dickson. The University of Chicago Press, 1930. 10+230 pp.

More than a hundred years have elapsed since Gauss laid the foundation of the theory of ternary quadratic forms, developed by him only so far as was necessary for his immediate purposes. At the end of that section of *The Disquisitiones* dealing with ternary forms, Gauss points out for future investigators an immense field for research; namely, the arithmetical investigation of forms of higher degrees as well as of quadratic forms with more than two variables.

Despite the efforts of the greatest mathematicians up to our day, very little has been achieved in regard to forms of higher degrees, even in the binary case. On the contrary, the development of the theory of quadratic forms with three and more variables, in the hands of men like Eisenstein, Hermite, Smith, Minkowski, A. Meyer, and Voronoi, not to speak of other important contributors in this field, was crowned with considerable success. Their work is scattered in different publications and therefore is not easily accessible. Moreover, due to the natural complexity of the matter, it needs careful revision, since even the greatest among mathematicians are not proof against errors, which are sometimes slight and sometimes more serious, as the case may be.

The first attempt to present the theory of quadratic forms with three or more variables in its essential parts was made by Bachmann in two valuable volumes published in 1898 and 1923. Paying all due respect to an easy presentation of difficult matters and to the variety of subjects included in Bachmann's work, one cannot refrain from feeling that there still remains much to be desired. The main criticism of Bachmann's work, in the opinion of the reviewer, is that it lacks the critical revision of results and the proofs found in the original papers; so it often happens that defects for which even authoritative investigators should be held responsible are reproduced in Bachmann's book without change. One must be always on guard when trying to use Bachmann's *Arithmetik der Quadratischen Formen* as a reference book in his own research.

Under such circumstances, those who cultivate the field of quadratic forms—and their number, fortunately, is increasing—cannot but welcome the valuable volume under review. The rather indefinite title does not reveal that the book deals almost exclusively with questions pertaining to the arithmetical theory of ternary and quaternary quadratic forms. It is by no means a treatise on quadratic forms, but a presentation of some important selected topics of which Professor Dickson has made a profound study, and which he has enriched by valuable personal contributions. The reviewer and many others would certainly like to see a complete treatise on quadratic forms of the same high qualities as Professor Dickson's book, but it seems the time is not ripe for such an arduous enterprise, inasmuch as many important parts in the theory of quadratic forms are still in a state of infancy; for instance, the reduction theory of the indefinite (even ternary) forms.

Although limited in scope, Professor Dickson's book presents exactly what we miss in Bachmann's book: the carefully revised work of his predecessors,